

MATH 247

Course Syllabus

Summer 2026

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Contact Information

Instructor

Name: Dr. Jonelle Hook

Contact Information

Email: jhook@msmary.edu

Phone: 301-447-5562

Office: COAD Science 156

Office Hours

Monday 9:00am - 10:00am

Wednesday, 9:30am - 10:30am

via [Zoom \(https://msmary.instructure.com/courses/22915/external_tools/10046\)](https://msmary.instructure.com/courses/22915/external_tools/10046)

Course Meeting Time

Course Format:

Online Asynchronous

For course format descriptions:

msmary.edu/course-formats  (<https://msmary.edu/course-formats>)

For student Zoom resources:

[Zoom Resources \(https://msmary.instructure.com/courses/2804/pages/zoom-resources-for-students\)](https://msmary.instructure.com/courses/2804/pages/zoom-resources-for-students)

Synchronous Location (if applicable):

Via [Zoom \(https://msmary.instructure.com/courses/22915/external_tools/10046\)](https://msmary.instructure.com/courses/22915/external_tools/10046) for the Midterm Exam and Final Exam

Course Description

An introduction to the fundamental concepts of differential and integral calculus with an emphasis on limits, continuity, derivatives and integrals of elementary functions. Applications to curve sketching, max-min values, related rates and areas will be given. Derivatives and integrals of elementary transcendental functions.

Learning Objectives



Course Learning Objectives

***Goal 1: Understand, analyze, and interpret concepts of calculus.***

To achieve this goal students will:

- Understand and use the definition of a limit.
- Understand concepts of continuity and differentiability.
- Understand the connection between integral calculus and differential calculus, The Fundamental Theorem of Calculus.
- Interpret a derivative in terms of slope and an integral in terms of area.

Goal 2: Develop mathematical skills to solve problems and communicate the results using calculus.

To achieve this goal students will:

- Calculate the derivative of a function using the definition and differentiation rules.
- Solve related rates and max-min problems by applying the methods of differentiation.
- Use properties of functions and aspects of calculus to interpret and sketch the graph of a curve.
- Graph, solve, differentiate, and integrate expressions involving exponential and logarithmic functions.

Math 247 satisfies the Department Goals for Mathematics majors listed below.

MA1 Proficiency in important areas of undergraduate mathematics such as analysis, algebra, and discrete mathematics.

MA2 The ability to investigate, clarify, and solve quantitative problems.

MA3 The ability to communicate mathematics with precision and clarity.

MA5 Preparation for success in graduate study and professional careers in business, industry, government, and teaching.

University Program Goals ▼

As a Catholic University grounded in the liberal arts, we ask all students to complete a common, sequenced, and interdisciplinary core curriculum. The University intends the whole of its undergraduate program to enable students to:

- The Catholic Vision of the Human Person: Understand and articulate the Catholic vision of the human person, particularly as it relates to the nature of the good, the relationship between faith and reason, and the human relationship with God;
- The Western Tradition: Integrate diverse modes of human inquiry and expression through rigorous study of the Western tradition, including its American expression;
- Competencies: Master the skills of analysis, interpretation, communication, and problem solving;
- Major Field of Study: Understand the purposes and concepts of at least one major field of study and become proficient in its methodology;
- Social Justice in a Global Community: Understand the diversity of human cultures in a global community, to see and seek to respond with justice and solidarity to all in the global community, to protect human dignity, to work for peace and freedom, and to respect the integrity of creation;
- A Life Well-Lived: Continue a life of learning, growth in faith and mature spirituality, and service to the common good.

Course Objectives	Math Program Goals	Undergraduate Program Goals
1	MA1, MA3, MA5	Competencies, Major Field of Study
2	MA1, MA2, MA3, MA5	Competencies, Major Field of Study

Textbooks & Reading Material

OpenStax Calculus Volume 1 (<https://openstax.org/details/books/calculus-volume-1>)

This is a free resource in which you can read the text online, download an app to view the text on a mobile device, or download a .pdf copy. If you prefer a hard copy of the text, you may purchase one either through the bookstore or using the link above.



Edfinity (https://msmary.instructure.com/courses/22915/external_tools/17369)

This is our online homework system. You will need to purchase access. The fee is approximately \$35.



Course Policies



Assignment Policies



Edfinity Homework

Homework will be assigned and graded using Edfinity. You can access Edfinity through Canvas by clicking on the link in the Navigation menu. Follow the instructions to either sign-in or create an account.

Homework will be assigned for each section of the textbook which we cover. The questions will be similar to those in the textbook. Assignments will overlap with three to five assignments per week. Assignments are due by 11:59pm on Tuesdays and Thursdays. Homework due dates for each section will be specified on Edfinity and in Canvas.

The point total for each homework will vary depending on the number of questions. You can submit each question of your homework up to 5 times for grading except multiple choice questions allow for 2 submissions.

Homework Solutions Uploaded to Canvas

In addition to completing the assignments on Edfinity, you are required to upload your complete handwritten solutions to each Edfinity Homework Assignment on Canvas. There will be an Assignment on Canvas titled "x.x - Upload your work." Click to open the Assignment and then click on Submit Assignment to upload your solutions. Each submission will be graded on a 3-point scale: 0 (no submission), 1 (incomplete submission), or 2 (complete submission).

For your course grade, the Homework average will be a weighted average of Edfinity scores (2/3) and handwritten solutions (1/3).

Canvas Quizzes

There will be four chapter quizzes on Canvas. They will be available for you to complete over a four-day period which opens on Friday morning and closes on Monday evening. Quizzes are timed (1 hour) and so you should ensure you have allotted the appropriate time to complete the quiz once you begin. You are allowed to use your text and notes for quizzes. You are not allowed to use any other persons or any internet resources such as search engines (Google, Bing, etc.), computational engines (WolframAlpha, Desmos, etc.), educational support systems (Chegg, Course Hero, etc.), or AI (ChatGPT, Bard, etc.).

Make Up Policy

If you miss a homework assignment, you can make it up as late work is enabled in Edfinity. You will receive up to 90% credit for work done up to 3 days past the due date.

In the event that you miss a chapter quiz or exam, the following rules hold: (1) severe personal illness or emergency, (2) the instructor must be notified as soon as possible preferably prior to missing the assessment, and (3) ample documentation of circumstances is required. Documentation includes a written explanation of your absence as well as official documentation i.e. doctor's note or athletic absence form. If the instructor accepts it as an excused absence, then accommodations will be made on a case-by-case basis.

Attendance/Participation ▼

You are required to participate in activities that further your development as an academic studying mathematics. For this class, you must earn **14 Academic Development (AD)** points to earn full credit. You can earn these points in a variety of ways. See [AD Points](https://msmary.instructure.com/courses/22915/pages/ad-points) (<https://msmary.instructure.com/courses/22915/pages/ad-points>) for details.

Grade Scale ▼

Your final course grade will be determined as follows.

Midterm Exam	25%
Final Exam	25%

Homework	20%
Quizzes	20%
Academic Development	10%

Grading scale:

Letter	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Ending %	100	93	89	86	82	79	76	72	69	66	62	59
Starting %	94	90	87	83	80	77	73	70	67	63	60	0

Student Responsibilities

Students are expected to

- read the textbook and course notes on Canvas.
- watch the course videos on Canvas.
- complete practice worksheets posted on Canvas and compare to solutions.
- utilize online textbook resources lecture videos, and interactive eBook links to video examples and demonstrations.
- check email, the Canvas course website, and Edfinity daily.
- work on homework assignments and quizzes.
- attend online office hours and email me as soon as help is needed.

During a regular semester, Calculus meets for 200 minutes a week. Students are expected to spend about 5 to 9 hours outside of class on homework and class prep. In total, that is 8 to 12 hours per week for 16 weeks. For a semester, that is 128 to 192 hours.

You should expect to spend that same amount of time on this course over our summer session. Our class is 8 weeks, this works out to about 16 to 24 hours per week.

Proctored Exams

There will be two exams on Canvas, a midterm exam and a final exam. They will be proctored using Zoom at a specified time slot on each day.

Midterm Exam: Thursday, June 18, 2026

Final Exam: Friday, July 17, 2026

Each exam is closed-book, but you are allowed to bring a two-sided 8.5 by 11 inch sheet of paper which may include definitions, formulae, examples, etc. This sheet will be turned in with the exam. You are also allowed to use a calculator. Cell phones and other devices are not permitted. Using outside sources/people and technology (Google, Wolfram Alpha, ChatGPT, etc.) during exams is a violation of the Mount St. Mary's University Standards on Academic Integrity.

AI Use Policy

There may be an AI Use Policy for student work and submissions in this course. If so, see below for important information on student AI use.

Assignments and Descriptions

Complete Assignment on Edfinity	Upload Written Solutions to Canvas	Due Date
Chapter 1 - Day 1	Ch 1 - Day 1 - Upload your work	5/28/26
Chapter 1 - Day 2	Ch 1 - Day 2 - Upload your work	5/28/26
2.2	2.2 - Upload your work	6/2/26
2.3	2.3 - Upload your work	6/2/26
2.4	2.4 - Upload your work	6/4/26
3.1	3.1 - Upload your work	6/4/26
Chapter 1 Quiz		6/8/26
3.2	3.2 - Upload your work	6/9/26
3.3	3.3 - Upload your work	6/11/26
3.4	3.4 - Upload your work	6/11/26

Complete Assignment on Edfinity	Upload Written Solutions to Canvas	Due Date
Chapter 2 Quiz		6/15/26
3.5	3.5 - Upload your work	6/16/26
3.6	3.6 - Upload your work	6/16/26
Midterm Exam		6/18/26
3.7	3.7 - Upload your work	6/23/26
3.8	3.8 - Upload your work	6/23/26
3.9	3.9 - Upload your work	6/23/26
4.2	4.2 - Upload your work	6/25/26
4.3	4.3 - Upload your work	6/25/26
Chapter 3 Quiz		6/29/26
4.4	4.4 - Upload your work	6/30/26
4.5	4.5 - Upload you work	6/30/26
4.6	4.6 - Upload your work	7/2/26
4.7	4.7 - Upload your work	7/2/26
4.8	4.8 - Upload your work	7/7/26
4.10	4.10 - Upload your work	7/7/26
5.1	5.1 - Upload your work	7/9/26
5.2	5.2 - Upload your work	7/9/26
Chapter 4 Quiz		7/13/26
5.3	5.3 - Upload your work	7/14/26
5.4	5.4 - Upload your work	7/14/26
5.5	5.5 - Upload your work	7/14/26
5.6	5.6 - Upload your work	7/16/26
5.7	5.7 - Upload your work	7/16/26
Final Exam		7/17/26

Course Calendar

Download the **DAILY SCHEDULE** (https://msmary.instructure.com/courses/22915/files/2868176?location=course_syllabus_22915&wrap=1) 
(https://msmary.instructure.com/courses/22915/files/2868176/download?location=course_syllabus_22915&download_frd=1) with homework due dates, quiz dates, and exam dates.

Technology Requirements

Technology tools and skills needed to be successful in your courses:

- Understand basic computer hardware and software
- Know how to use a keyboard and a mouse
- Be able to manage files and folders using operations such as save, copy, rename, delete etc.
- Be competent in the use of Microsoft Office or applications similar to Word, Excel, PowerPoint
- Be able to send and download files and attachments.
- Know how to communicate using live.msmmary.edu email
- Know how to use the Internet to access your course, perform online research, and use search engines and databases.

Technology Requirements:

Please review the Information Technology Support Center's (ITSC) [**minimum software and hardware requirements**](#) 

(<https://msmaryedu.sharepoint.com/ITSC/Shared%20Documents/Laptop%20Specifications.pdf>).

Please note that courses often cannot be successfully completed on a Chromebook, tablet, or cellular device. Consistent access to a reliable computer and Internet connection are required for all courses.

Additional information on using Canvas, Microsoft Office, and web conferencing tools can be found on the "Student Resources" link available from the course home page.

If students have technical difficulties submitting assignments in Canvas, they should contact Canvas Help for assistance *before* the due date/time. Representatives from Canvas will provide technical support and an email confirmation regarding the status of the request. Students must forward the email to the professor along with their assignment to earn full credit if the problem cannot be resolved prior to the deadline.

If you have computer issues or are unable to access your Mount e-mail contact ITSC at 301-447-5805 or submit a support ticket by emailing itsc@msmary.edu (<mailto:itsc@msmary.edu>).

Mount Policies & Info

Academic Integrity ▼

An academic community, regardless of delivery modality (e.g., in-person, online), must operate with complete openness, honesty and integrity. Responsibility for maintaining this atmosphere lies with the students, faculty and administration. Therefore, the achievement of personal and academic goals through dishonest means will not be tolerated. Academic misconduct includes but is not limited to:

1. **Cheating:** the unauthorized use, access, or exchange of information before or during a quiz, test or semester examination. Unauthorized collaboration on a class assignment, submitting the same work in two courses without the professor's permission, and buying or selling work for a course are also forms of cheating.
2. **Plagiarism:** the representation of someone else's words or ideas as one's own. The various forms of plagiarism include but are not limited to copying homework, falsifying lab reports, submitting papers containing materials written by another person, and failing to document correctly in one's written assignment words, arguments or ideas secured from other sources including content available from websites, and/or generated through A.I. without the instructor's permission. Plagiarism detection software may be used with assignment submissions to verify original work.

3. **Providing or receiving assistance in a manner not authorized by the professor** in the creation of work to be submitted for academic evaluation including papers, projects and examinations; presenting as one's own the ideas or words of another (including those generated from A.I.) for academic evaluation without proper acknowledgement and permission from the instructor.
4. **Doing unauthorized academic work** for which another person will receive credit or be evaluated.
5. **Misrepresenting student identity:** posing as another or having someone pose as you is an act of academic misconduct. Identity verification may be required to complete course activities and assessments.
6. **Attempting to influence one's academic evaluation** by means other than academic achievement or merit including unauthorized access to computer systems.
7. **Assisting in misconduct:** cooperation with another in an act of academic misconduct. A student who writes a paper, posts a paper or exam questions online, or does an assignment for another student is an accomplice and will be held accountable just as severely as the other. Any student who permits another to copy from his or her own paper, examination, or project shall be held as accountable as the student who submits the copied material. Students are expected to safeguard their work and should not share papers, projects, homework, or exam answers with other students unless specifically directed to by their professors.
8. **Failure to protect University accounts and computer systems** such as sharing of accounts, passwords, and other assigned resources whether with intention or through negligence.

Canvas Information

Canvas is where course content, grades, and communication will reside for this course. Should you need assistance using Canvas click on the "Help" icon on the far left of your screen at the bottom of your blue navigation menu. From that link you will have access to:

- A Frequently Asked Questions (FAQs) document with direct links to support
- Canvas Guides for Students

- Live chat with Canvas Help Desk (24/7). MSMU has the premium subscription. Please don't hesitate to contact them for help.
- Live phone support, also 24/7
- For Canvas passwords or any other computer-related technical support contact the [ITSC Help Desk](https://inside.msmary.edu/more/information-technology/itsc.html)  (<https://inside.msmary.edu/more/information-technology/itsc.html>).
 - ITSC@msmary.edu (<mailto:ITSC@msmary.edu>)
 - 301-447-5805

Classroom Accommodations for Students with Disabilities

If you are a student with a documented disability who requires an academic adjustment, auxiliary aid, or similar accommodation, please contact Mount St. Mary's Learning Service director to discuss eligibility for accommodations and academic adjustments at 301-447-5006 (V/TTD).

Mount Report: Campus-wide Reporting

In the spirit of providing students a quality education and ensuring student success, offering an appropriate array of support services, and supporting student concerns, Mount St. Mary's University takes seriously concerns and complaints brought to its attention.

As such when a student encounters a problem on campus or feels they have been treated unfairly, the student should first attempt to resolve the issue informally with the employee, department, service, process or administrative procedure involved. Many issues can be resolved by making an appointment and honestly communicating the concern(s).

If a student is not satisfied after working to resolve the matter directly, they may use the [Mount Report](https://msmary.edu/reporting.html)  (<https://msmary.edu/reporting.html>) process to file a formal student complaint.

[Mount Report Link](https://msmary.edu/reporting.html)  (<https://msmary.edu/reporting.html>)

The Student Complaint Process does not apply to grade appeals, complaints of sexual harassment, or any student-to-student complaints.

The Mount Report process can be used for reporting violations of Title IX policy*. Please refer to the [Mount St. Mary's University Catalog](http://catalog.msmary.edu/) , [\(http://catalog.msmary.edu/\)](http://catalog.msmary.edu/), [Policy on Title IX Sexual Harassment](https://msmary.edu/title-ix-sexual-harassment-discrimination-policy/index.html) , [policy/index.html](https://msmary.edu/title-ix-sexual-harassment-discrimination-policy/index.html)), or [Student Code of Conduct](https://inside.msmary.edu/student-affairs/files/msmu-student-code-of-conduct.pdf) , [conduct.pdf](https://inside.msmary.edu/student-affairs/files/msmu-student-code-of-conduct.pdf)), respectively, for procedures regarding these types of complaints.

Intellectual Property

The use of copyrighted materials in this course to support teaching and learning is done so with permission from the copyright holder or according to Fair Use Guidelines and the TEACH Act. As such, you are permitted to download a copy of copyrighted works posted in our secured course site in Canvas for your personal use, but you are not authorized to redistribute to others (e.g., make copies to distribute, forward as attachments). Similarly, the intellectual property of students (e.g., presentations/posts in discussion forums) should not be shared with or redistributed to others.

Netiquette

Please consider these guidelines as you actively participate in our course.

Be constructive: Our goal is to develop a safe, collaborative, and constructive learning community. Constructive posts are encouraged. Often sharing a differing perspective or opinion advances our shared learning. When responding to a classmate, respectfully indicate why you do or do not agree with their view. If you would not make a comment face-to-face, please do not do so online. Refrain from using any offensive or foul language, always discuss or respectfully

debate content, but never attack a member of our learning community. Try not to dominate any discussion, but do regularly share your experiences and knowledge.

Be considerate: Though mobile technologies have enhanced our ability to access course content from anywhere and at any time, we should not expect instantaneous responses to our posts or questions. Give others a reasonable amount of time to respond within the participation guidelines established by the course instructor. Once you have posted or replied in a discussion thread, regularly check the thread for new comments, and respond accordingly.

Be professional: Often in the online environment, facial expressions and body language are absent and what you intended to be constructive may be interpreted otherwise. In addition to spelling and grammar considerations, you will want to read your comments before you post to ensure that your intent is accurately conveyed. Often a bit of politeness goes a long way, so if someone helps out, thank that person online just as you would face-to-face. Refrain from using all caps as that is perceived as shouting in the online setting. If you receive a private message, reply in private. There may be a reason the other participant chose to comment in private. You should not forward to another or post publicly.

Be respectful: This institution and its instructors embrace learners from diverse backgrounds and seeks to develop and support an inclusive learning community. Please be sensitive to the diversity of all members in our learning community and ensure your comments are inclusive to all members. Learning is about sharing ideas and concepts. We do not all have to agree with an opinion or comment, but we should be open to and respectful of others' ideas and comments. MSMU supports open discussion, but does not tolerate sexist, bigoted, racist, or demeaning content. Your instructor can and will censor or remove a post if he/she deems it violates institutional guidelines or course expectations. You will be contacted in the event of any such censorship to identify how the content violated guidelines and to discuss supportive roles in our learning community.

Be appropriate: When participating in synchronous conferences or posting video to the LMS remain professional and appropriate in your setting and in your attire at all times.

Stay positive: Keeping a positive attitude can go a long way to building a supportive learning community and toward reducing personal stress. If you are having any difficulties or concerns with the course (e.g., using Canvas, understanding an assignment or activity, comfort with a discussion) please contact your instructor or a peer for support. Often early communication and requests for assistance can alleviate larger issues later.

Privacy Policy

To ensure your privacy and security with communications, assignment submissions, and assessment feedback, only the Canvas Learning Management System (course communications, grading) and MSMU email (other written communication) will be used.

COURSE

[MATH 247 A: Calculus I \(Summer 2026\)](#)

 [Assignments](#)

 [Calendar](#)

 [Course information](#)



[My Courses](#) / MATH 247 A: Calculus I (Summer 2026)

Assignments

Search assignments



All assignments

29 assignments

Name ^	Dates ^	Score % ^
 Chapter 1 - Day 1 homework • completed	May 28, 2026 11:59 PM: due May 30, 2026 11:59 PM: late period ends	100%
 Chapter 1 - Day 2 homework • completed	May 28, 2026 11:59 PM: due May 30, 2026 11:59 PM: late period ends	100%
 2.2 The Limit of a Function homework • completed	Jun 02, 2026 11:59 PM: due Jun 04, 2026 11:59 PM: late period ends	100%
 2.3 The Limit Laws homework • completed	Jun 02, 2026 11:59 PM: due Jun 04, 2026 11:59 PM: late period ends	100%
 2.4 Continuity homework • completed	Jun 04, 2026 11:59 PM: due Jun 06, 2026 11:59 PM: late period ends	100%
 3.1 Defining the Derivative homework • completed	Jun 04, 2026 11:59 PM: due Jun 06, 2026 11:59 PM: late period ends	100%
 3.2 The Derivative as a Funct... homework • completed	Jun 09, 2026 11:59 PM: due Jun 11, 2026 11:59 PM: late period ends	100%
 3.3 Differentiation Rules homework • completed	Jun 11, 2026 11:59 PM: due Jun 13, 2026 11:59 PM: late period ends	100%
 3.4 Derivatives as Rates of C... homework • completed	Jun 11, 2026 11:59 PM: due Jun 13, 2026 11:59 PM: late period ends	100%
 3.5 Derivatives of Trigonome... homework • completed	Jun 16, 2026 11:59 PM: due Jun 18, 2026 11:59 PM: late period ends	100%
 3.6 The Chain Rule assignment • completed	Jun 16, 2026 11:59 PM: due Jun 18, 2026 11:59 PM: late period ends	100%
 3.7 Derivatives of Inverse Fu... homework • completed	Jun 23, 2026 11:59 PM: due Jun 25, 2026 11:59 PM: late period ends	100%
 3.8 Implicit Differentiation homework • completed	Jun 23, 2026 11:59 PM: due Jun 25, 2026 11:59 PM: late period ends	80%
 3.9 Derivatives of Exponentia... homework • completed	Jun 23, 2026 11:59 PM: due Jun 25, 2026 11:59 PM: late period ends	92%

[Help](#)

Grades

Course information

100%

MATH 247 - Calculus 1

Section 4.7: Optimization

Guidelines for Solving Optimization Problems

1. **Read and understand the problem.**
 - Determine known and unknown quantities.
2. **Draw a diagram.**
3. **Assign symbols to the quantities.**
 - Equation A : To maximize or minimize.
 - Equation B : Equal to some constant.
 - Solve B for one of the variables.
 - Plug into A so that A is a single-variable function.
 - Find the derivative of A .
4. **Use the First Derivative Test.**
 - Suppose c is the only critical number for $f(x)$.
 - If $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$, then c is an absolute max.
 - If $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$, then c is an absolute min.
 - Test endpoints, if any.

Example. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

Example. What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

Example. Minimize the surface area of a cylinder with volume 1000 cm^3 .

Example. Find two nonnegative numbers whose sum is 9 such that the product of one number and the square of the other number is a maximum.

Example. Find the point (x, y) on the graph of $y = \sqrt{x}$ nearest the point $(4, 0)$.

Example. A container in the shape of a right circular cylinder with no top has surface area 3π ft². What height h and base radius r will maximize the volume of the cylinder?

Example. A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

MATH 247 - Calculus 1

Section 4.2: Linear Approximations and Differentials

Definition. The linear approximation or tangent line approximation of $f(x)$ at $x = a$ is the linear function

$$L(x) = f'(a)(x - a) + f(a).$$

Example. Find the linear approximation of $f(x) = \ln x$ at $x = 1$, then use it to approximate $\ln(1.05)$.

Example. Use a linear approximation to estimate the value of $(16.034)^{3/2}$.

Example. Find the linear approximation of $f(x) = \sqrt{x+3}$ at $x = 1$, then use it to approximate $\sqrt{3.98}$.

Example. Use a linear approximation to estimate the value of $(2.001)^5$.

MATH 247 - Calculus 1

Section 4.3: Maxima and Minima

Definition. We define the following extreme values.

A function $f(x)$ has an **absolute maximum** at $x = c$ if $f(c) \geq f(x)$ for all x in the domain. The **absolute maximum value** is $f(c)$.

A function $f(x)$ has an **absolute minimum** at $x = d$ if $f(d) \leq f(x)$ for all x in the domain. The **absolute minimum value** is $f(d)$.

A function $f(x)$ has a **local maximum** at $x = c$ if $f(c) \geq f(x)$ when x is near c . The **local maximum value** is $f(c)$.

A function $f(x)$ has a **local minimum** at $x = d$ if $f(d) \leq f(x)$ when x is near d . The **local minimum value** is $f(d)$.

Example. Draw the graph and label the extreme values.

Theorem (Extreme Value Theorem). If f is continuous on $[a, b]$, then f attains an absolute maximum value at a point in $[a, b]$ and an absolute minimum value at a point in $[a, b]$.

Draw the three example graphs.

Why continuous?

Why closed interval?

Theorem (Fermat's Theorem). If f has a local maximum or local minimum at $x = c$ and $f'(c)$ exists, then $f'(c) = 0$.

WARNINGS:

1.

2.

Fermat's Theorem says:

Definition. The function f has a **critical number** c if $f'(c) = 0$ or $f'(c)$ does not exist.

We call the point $(c, f(c))$ a **critical point** of f .

Example. Find the critical numbers of $f(x) = \frac{x-1}{x^2-x+1}$.

Closed Interval Method

Find **absolute** max and min values of a continuous function f on a closed interval $[a, b]$:

1. Find all critical numbers in (a, b) .
2. Find the function values of all critical numbers and endpoints of the interval

Example. Find the absolute max and min of $y = x\sqrt{4 - x^2}$ on $[-1, 2]$.

Example. Find the local and absolute extreme value of $f(x) = \cos x$.

Example. Find the local and absolute extreme value of $f(x) = x^2$.

Example. Find the local and absolute extreme value of $f(x) = x^3$.

Example. Use the graph to find the local and absolute extreme value of $f(x) = 3x^4 - 16x^3 + 18x^2$ on $[-1, 4]$.

Example. Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

Example. Find the critical numbers of $g(x) = \frac{x-3}{x^2-x+3}$.

Example. Find the absolute max and min of $f(x) = x^3 - 3x^2 + 1$ when $-1/2 \leq x \leq 4$.

Example. Find the absolute max and min of $f(x) = \frac{x}{x^2 + 4}$ on $[0, 3]$.

MATH 247 - Calculus 1

Section 4.4: The Mean Value Theorem

Theorem (Rolle's Theorem). If f is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$, then there is some c in (a, b) such that $f'(c) = 0$.

Example. Verify the function $f(x) = x^3 - x^2 - 6x + 2$ over $[0, 3]$ satisfies the hypothesis of Rolle's Theorem and find all numbers that satisfy the conclusion of Rolle's Theorem.

Theorem (The Mean Value Theorem). If f is continuous on $[a, b]$ and differentiable on (a, b) , then there is some c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The Mean Value Theorem (MVT) says:

There is a number where the instantaneous rate of change equals the average rate of change.

Example. Verify the function $f(x) = \frac{1}{x}$ over $[1, 3]$ satisfies the hypothesis of the Mean Value Theorem and find all numbers that satisfy the conclusion of the Mean Value Theorem.

Example. If $f(0) = 3$ and $f'(x) \leq 5$ for all x , how large can $f(2)$ be?

Example. Verify the function $f(x) = 5 - 12x + 3x^2$ over $[1, 3]$ satisfies the hypothesis of Rolle's Theorem and find all numbers that satisfy the conclusion of Rolle's Theorem.

Example. Show that $f(x) = x^3 + x - 1$ has exactly one real root.

Example. Verify the function $f(x) = x^2 - x$ over $[0, 2]$ satisfies the hypothesis of the Mean Value Theorem and find all numbers that satisfy the conclusion of the Mean Value Theorem.

Examples. For the following functions, show there is no c that satisfies the conclusion of the Mean Value Theorem over $[-1, 1]$.

(a) $f(x) = |x - \frac{1}{2}|$

(b) $f(x) = \frac{1}{x^2}$

Example. Determine whether the Mean Value Theorem applies for the functions below over the given interval. If so, find the value(s) that satisfy the conclusion of the theorem. If not, explain why.

(a) $y = e^x$ over $[0, 1]$

(b) $y = \sqrt{9 - x^2}$ over $[-3, 3]$

(c) $y = \frac{x}{x+1}$ over $[-2, 2]$

(d) $y = \frac{x}{x+1}$ over $[0, 2]$

Theorem. If $f'(x) = 0$ for all x in (a, b) , then $f(x)$ must be constant on (a, b) .

Corollary. If $f'(x) = g'(x)$ for all x in (a, b) , then $f(x) = g(x) + C$ where C is a constant.

MATH 247 - Calculus 1

Section 4.5: Derivatives and Shapes of Graphs

Theorem (First Derivative Test). Suppose that f is continuous over an interval I containing a critical point c . If f is differentiable over I , except possibly at point c , then $f(c)$ satisfies one of the following:

- (i) If f' changes sign from positive when $x < c$ to negative when $x > c$, then $f(c)$ is a local maximum of f .
- (ii) If f' changes sign from negative when $x < c$ to positive when $x > c$, then $f(c)$ is a local minimum of f .
- (iii) If f' has the same sign for $x < c$ and $x > c$, then $f(c)$ is neither a local maximum nor a local minimum.

Definition. A graph is **concave up** if the graph lies above all its tangent lines.

A graph is **concave down** if the graph lies below all its tangent lines.

Definition. An **inflection point** is a point on the curve where the graph changes in concavity.

Theorem (Concavity Test). Let f be a function that is twice differentiable over an interval I .

- (a) If $f''(x) > 0$ for all $x \in I$, then f is concave up over I .
- (b) If $f''(x) < 0$ for all $x \in I$, then f is concave down over I .

Example. Find the intervals of $f(x) = 2x^3 + 3x^2 - 36x$ where $f(x)$ is increasing or decreasing with local extreme values. Find the intervals of concavity with inflection points.

Example. Find the intervals of $f(x) = x^{1/3}(x + 4)$ where $f(x)$ is increasing or decreasing with local extreme values. Find the intervals of concavity with inflection points.

Example. Sketch the graph of a function that satisfies:

$$f'(x) > 0 \text{ on } (-\infty, 1)$$

$$f'(x) < 0 \text{ on } (1, \infty)$$

$$f''(x) > 0 \text{ on } (-\infty, -2) \cup (2, \infty)$$

$$f''(x) < 0 \text{ on } (-2, 2)$$

$$\lim_{x \rightarrow -\infty} f(x) = -2$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

Example. Find where $f(x) = 2x^4 - \frac{16}{3}x^3 - 12x^2 + 10$ is increasing and decreasing.

Example. Find the local max and min values of $g(x) = x + \sqrt{2} \cos x$ where $0 \leq x \leq \pi$.

Example. Sketch the graph of a function that satisfies:

$$f'(x) > 0 \text{ on } (2, \infty)$$

$$f'(x) < 0 \text{ on } (-\infty, 2)$$

$$f''(x) > 0 \text{ on } (-1, 1) \quad f''(x) < 0 \text{ on } (-\infty, -1) \cup (1, \infty)$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = 10$$

Example. For $f(x) = x^3 - 6x$, find the intervals of increase or decrease along with local minimum and maxima of f . Then find the intervals of concavity with inflection points.

Example. For $f(x) = \frac{1}{x-1}$, find the intervals of increase or decrease along with local minimum and maxima of f . Then find the intervals of concavity with inflection points.

Example. For $f(x) = \ln(x^4 + 27)$, find the intervals of increase or decrease along with local minimum and maxima of f . Then find the intervals of concavity with inflection points.

MATH 247 - Calculus 1

Section 4.6: Curve Sketching

Guidelines for Curve Sketching

1. Domain.

- Denominators - exclude values that make the denominator zero.
- Square roots - include values where the radicand is greater than or equal to zero.
- Logarithms - include values where the argument is greater than zero.

2. Intercepts.

- Let $y = 0$ and find the x-intercept.
- Let $x = 0$ and find the y-intercept.

3. Asymptotes.

- Vertical asymptotes - find values with an infinite limit.
- Horizontal asymptotes - check limits as x approaches ∞ and $-\infty$.

4. Increasing/Decreasing Test and Local Extreme Values.

- $f'(x) > 0 \Rightarrow f(x)$ increasing
- $f'(x) < 0 \Rightarrow f(x)$ decreasing
- $f'(x)$ changes from positive to negative $\Rightarrow$ local max
- $f'(x)$ changes from negative to positive $\Rightarrow$ local min

5. Concavity and Inflection Points.

- $f''(x) > 0 \Rightarrow f(x)$ concave up
- $f''(x) < 0 \Rightarrow f(x)$ concave down
- change in concavity $\Rightarrow$ inflection point

6. Sketch the graph.

- Be sure to use the original function to plot any extreme values and inflection points.

Example. Use the guidelines to sketch $y = \frac{2x^2}{x^2 - 1}$.

Example. Use the guidelines to sketch $y = x^4 - 2x^3 + 3$.

Example. Use the guidelines to sketch $y = \frac{-3x^2}{x^2 - 4}$.

MATH 247 - Calculus 1

Section 4.8: L'Hôpital's Rule

Definition. An **indeterminate form** occurs when evaluating a limit gives an expression whose value cannot be determined directly because it could lead to different outcomes depending on how the variables approach their limits.

Consider $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$.

If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$, then we call this an indeterminate form of **type** $0/0$.

If $\lim_{x \rightarrow a} f(x) = \infty$ and $\lim_{x \rightarrow a} g(x) = \infty$, then we call this an indeterminate form of **type** ∞/∞ .

Theorem (L'Hôpital's Rule). Suppose f and g are differentiable functions over an open interval containing a , except possibly at a . If we have an indeterminate form of type $0/0$ or ∞/∞ , then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \stackrel{H}{=} \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

Notes:

1.

2.

3.

Example. Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$.

Example. Evaluate $\lim_{x \rightarrow \infty} \frac{e^x}{x^2}$.

Example. Evaluate $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}}$.

Indeterminate Products Type $0 \cdot \infty$

Consider $\lim_{x \rightarrow a} f(x)g(x)$ where one function approaches zero and the other function approaches infinity as x approaches a .

Technique: Rewrite $f(x)g(x)$ as a quotient.

$$\frac{f(x)}{1/g(x)} \quad \text{or} \quad \frac{g(x)}{1/f(x)}$$

Example. Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

Indeterminate Differences Type $\infty - \infty$

Consider $\lim_{x \rightarrow a} f(x) - g(x)$ where both functions approach infinity as x approaches a .

Technique: Convert difference into a quotient by

- finding a common denominator,
- rationalization,
- factoring, or
- using a trig identity.

Example. Evaluate $\lim_{x \rightarrow \pi/2^-} \sec x - \tan x$.

Example. Evaluate $\lim_{x \rightarrow \infty} xe^{1/x} - x$.

Indeterminate Powers Type 0^0 , ∞^0 , 1^∞

Consider $\lim_{x \rightarrow a} f(x)^{g(x)}$.

Technique: Let $y = f(x)^{g(x)}$ and use natural log.

- Take the natural log of both sides: $\ln y = \ln(f(x)^{g(x)})$.
- Simplify: $\ln y = g(x) \cdot \ln(f(x))$.
- Find the limit of $\ln y$.
- Remember to “ e ” your answer.

Example. Evaluate $\lim_{x \rightarrow 0^+} x^x$.

Example. Evaluate $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$.

Example. Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$.

Example. Evaluate $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$.

Example. Evaluate $\lim_{x \rightarrow \infty} \frac{2x + 1}{3x^2 - 5}$.

Example. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{x}$.

Example. Evaluate $\lim_{x \rightarrow \pi^-} \frac{\sin x}{1 - \cos x}$.

Example. Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$.

Example. Evaluate $\lim_{x \rightarrow 0} \frac{1}{x^2} \cdot e^{-1/x^2}$.

Example. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} + \frac{5}{x^2}\right)^x$.

MATH 247 - Calculus 1

Section 4.10: Antiderivatives

Given a derivative $f'(x)$, what was the original function $f(x)$?

Example. $f'(x) = x^2$

Opposite of the Power Rule

$$f'(x) = x^n \text{ for } n \neq -1$$

$$f(x) = \frac{1}{n+1} x^{n+1}$$

“Raise it, flip it”

Example. $f'(x) = x^8$

Example. $f'(x) = x^{-4}$

Example. $f'(x) = \frac{5 + 2x - x^9}{x^7}$

Function	Antiderivative
$cf'(x)$	
$f'(x) \pm g'(x)$	
x^n for $n \neq 1$	
$\cos x$	
$\sin x$	
$\sec^2 x$	
$\sec x \tan x$	
e^x	
$\frac{1}{x}$	
$\frac{1}{1 + x^2}$	

Example. $f'(x) = 4 \cos x + \frac{3x^7 - 2\sqrt{x}}{x}$

Example. $f'(x) = 5x\sqrt[3]{x}$

Example. $f'(x) = x\sqrt{x}, f(1) = 2$

Example. $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, $f(1) = 1$

Definition. A function F is an **antiderivative** of f if $F'(x) = f(x)$.

(Reverse of Differentiation)

Theorem. If F is an antiderivative of f , then the most general antiderivative of f is

$$F(x) + C$$

where C is an arbitrary constant.

Example. $f'(x) = e^x + x^{-3} - \sqrt{x}$

Example. $f'(x) = 8x^7 - 7 \sec^2 x$

Example. $f'(x) = 7x^{2/5} + 8x^{-4/5}$

Example. $f'(x) = \frac{3 - 4x^3 + x^7 + 2x^8}{x^8}$

Example. $f''(x) = x^6 - 4x^4 + x + 1$

Example. $f'(x) = 5x^4 - 3x^2 + 4$, $f(-1) = 2$

Example. $f''(x) = \frac{3}{\sqrt{x}}$, $f(4) = 20$, $f'(4) = 7$

Example. $g'(x) = 4 \sin x + \frac{2x^5 - \sqrt{x}}{x}$

Example. A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 feet above ground. Find its height above ground t seconds later. When does it reach maximum height? When does it hit the ground?

Example. $f'''(x) = 8e^x - \sin x + x$

Example. $f'(x) = \frac{1}{\sqrt{x}} + \sec x \tan x - \frac{x^{1/3}}{x^{2/3}} - 8$

Example. Determine whether the following implication is true or false. Either prove it is true or find a counterexample if it is false.

If $f(x)$ is the antiderivative of $g(x)$, then $f(x) + 1$ is the antiderivative of $g(x) + 1$.

MATH 247 - Calculus 1

Chapter 4: Review for Quiz 3

The quiz material consists of Chapter 4 (4.1 through 4.8) and assumes the material from Chapters 1, 2, and 3. You are responsible for all assigned homework problems, worksheets, and topics/examples discussed in lecture and in videos. The questions below are a sampling of problems for review for the quiz. This sample is not complete, and is not representative of the quiz questions. Be sure to prepare all material from class, Canvas, and Edfinity.

1. Find the linear approximation of $f(x) = x \sin x$ at $a = 2\pi$.
2. Find the absolute maximum and absolute minimum values of $y = x^3 - 3x^2$ on $[-1, 3]$.

3. Verify that the function $f(x) = x^4 - 2x^2$ on the interval $(-2, 2)$ satisfies the hypotheses of Rolle's Theorem. Find all numbers c that satisfy the conclusion of Rolle's Theorem.

4. Verify that the function $f(x) = x^3 - 2x + 1$ on the interval $[-2, 3]$ satisfies the hypotheses of the Mean Value Theorem. Find all numbers c that satisfy the conclusion of the MVT.

5. Use the guidelines to sketch the following curves:

(a) $f(x) = x^3 - 6x^2 + 9x + 1$

(b) $f(x) = \frac{x}{x^2 - 1}$

(c) $f(x) = \frac{x-3}{x+3}$

6. A cardboard box has a base length that is three times the base width. If the box has a volume of 50 cubic feet, determine the dimensions of the box that will minimize the amount of material used to construct the box.
7. Construct a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area?

8. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
9. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.

10. Find two positive integers such that the sum of the first number and four times the second larger is 1000 and the product of the numbers is as large as possible.
11. The volume of a cube is increasing at a rate of $10 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30 cm?

12. Find the following limits, if they exist:

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3}$

(b) $\lim_{h \rightarrow 0} \frac{(h - 1)^3 + 1}{h}$

(c) $\lim_{x \rightarrow 1} \left(\frac{1}{x - 1} + \frac{1}{x^2 - 3x + 2} \right)$

(d) $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$

(e) $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$

(f) $\lim_{x \rightarrow \infty} (e^x + x)^{1/x}$